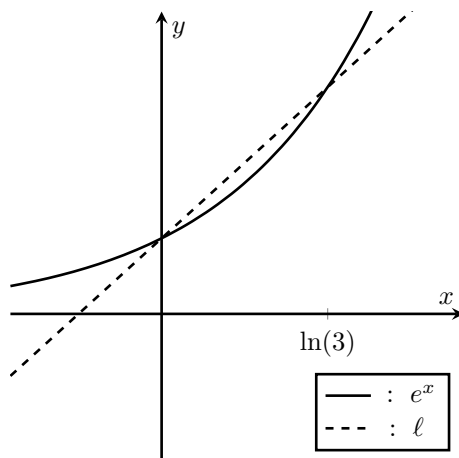


Examples of Homework Solutions

You are expected to write up complete, legible, logical solutions to all written homework problems assigned. Each solution should be written using complete sentences (English or mathematical) to explain your steps. Some assignments will contain drill exercises, for which you may just show your calculations and an answer (see the second Example #2 on this sheet). Each graded solution will be evaluated out of 3 points.

1. Find the equation of the line ℓ in the following graph:



The following is an example of a great solution to the above problem (such a solution would earn you 3 points):

- Because ℓ and the curve given by $y = e^x$ have the same y -intercept, we know that the y -intercept of the line ℓ is $(0, e^0) = (0, 1)$. So, we only need one other point on ℓ in order to determine an equation for the line. Observe that ℓ intersects the curve $f(x) = e^x$ at $x = \ln(3)$. In other words, the point $(\ln(3), e^{\ln(3)}) = (\ln(3), 3)$ lies on ℓ . We may then find the slope of ℓ :

$$m = \frac{3 - 1}{\ln(3) - 0} = \frac{2}{\ln(3)}.$$

Using slope-intercept form, an equation for ℓ is given by

$$y = \frac{2}{\ln(3)}x + 1.$$

Here are some examples of (correct) solutions that would **not** earn you full credit on this problem (the first two would each earn you 1 point because the explanation was incomplete or unclear, and the third would earn you 2 points, because it was not simplified):

- $y = \frac{2}{\ln(3)}x + 1$.
- We have $(0, 1)$ and $(\ln(3), 3)$, so $y = \frac{2}{\ln(3)}x + 1$.

- We know that the y -intercept of the line ℓ is $(0, e^0)$. So, we only need one other point on ℓ in order to determine an equation for the line. ℓ intersects the curve $f(x) = e^x$ at $x = \ln(3)$. In other words, the point $(\ln(3), e^{\ln(3)})$ lies on ℓ . We may then find the slope of ℓ :

$$m = \frac{e^{\ln(3)} - e^0}{\ln(3) - 0} = \frac{e^{\ln(3)} - e^0}{\ln(3)}.$$

Using slope-intercept form, an equation for ℓ is given by

$$y = \frac{e^{\ln(3)} - e^0}{\ln(3)}x + 1.$$

2. Suppose that $f(x) = x^3$, and $g(x) = \sqrt{x+8}$. Evaluate the following:

- $f(g(1))$
- $g(f(1))$
- $f(g(x))$
- $g(f(x))$
- $f(t)g(t)$

The following are examples of great solutions to the above problem (each would earn you 3 points):

- a) Since $g(1) = \sqrt{1+8} = \sqrt{9} = 3$, $f(g(1)) = f(3)$. Then, $f(3) = (3)^3 = 27$, and so $f(g(1)) = 27$.
 - b) Since $f(1) = (1)^3 = 1$, $g(f(1)) = g(1)$. Then, $g(1) = 3$, as shown in part a), and so $g(f(1)) = 3$.
 - c) Since $g(x) = \sqrt{x+8}$, $f(g(x)) = f(\sqrt{x+8})$. Then, $f(g(x)) = (\sqrt{x+8})^3$ (this may also be written as $(x+8)^{3/2}$).
 - d) Since $f(x) = x^3$, $g(f(x)) = g(x^3)$. Then, $g(f(x)) = \sqrt{x^3+8}$.
 - e) Since $f(t) = t^3$ and $g(t) = \sqrt{t+8}$, $f(t)g(t) = t^3\sqrt{t+8}$.
- a) $f(g(1)) = f(3) = 27$.
 - b) $g(f(1)) = g(1) = 3$.
 - c) $f(g(x)) = f(\sqrt{x+8}) = (\sqrt{x+8})^3$.
 - d) $g(f(x)) = g(x^3) = \sqrt{x^3+8}$.
 - e) $f(t)g(t) = t^3\sqrt{t+8}$.

Here is an example of a (correct) solution that would **not** earn you full credit on this problem (you would earn 1 point for such a solution, because no work was shown):

- a) 27
- b) 3
- $(\sqrt{x+8})^3$
- $\sqrt{x^3+8}$
- $t^3\sqrt{t+8}$